Problem Set #1: Calculus and Micro Theory

1. Explain intuitively the importance of taking derivatives and setting them equal to zero.

2. Use the definition of a derivative to prove that constants pass through derivatives, i.e., that $\frac{d}{dx}[(c \cdot f(x)] = c \cdot \frac{d}{dx}[f'(x)]$.

3. Use the product rule to prove that the derivative of x^2 is $2x$. (*Challenge*: Do the same for higher-order integer powers, e.g., x^{30}. *Do not* do this the hard way.)

4. For each of the following functions, calculate the first derivative, the second derivative, and determine maximum and/or minimum values (if they exist):

   (a) $x^2 + 2$

   (b) $(x^2 + 2)^2$

   (c) $(x^2 + 2)^{\frac{1}{2}}$

   (d) $-x(x^2 + 2)^{\frac{1}{2}}$

   (e) $\ln\left[(x^2 + 2)^{\frac{1}{2}}\right]$

5. Calculate partial derivatives with respect to x and y of the following functions:

   (a) $x^2 y - 3x + 2y$

   (b) e^{xy}

   (c) $e^x y^2 - 2y$

6. Imagine that a monopolist is considering entering a market with demand curve $q = 20 - p$. Building a factory will cost F, and producing each unit will cost 2 so its profit function (if it decides to enter) is $\pi = pq - 2q - F$.

   (a) Substitute for p using the inverse demand curve and find the (interior) profit-maximizing level of output for the monopolist. Find the profit-maximizing price and the profit-maximizing profit level.

   (b) For what values of F will the monopolist choose not to enter the market?

7. (Profit maximization for a firm in a competitive market) Profit is $\pi = p \cdot q - C(q)$. If the firm is maximizing profits and takes p as given, find the necessary first order condition for an interior solution to this problem, both in general and in the case where $C(q) = \dfrac{1}{2}q^2 + 2q$.