Problem Set #2: Expected Value and Insurance

1. An individual with initial wealth of $400 has a 20% chance of getting in an accident. If he gets in an accident, he will lose $300, leaving him with $100; if he doesn’t, he loses nothing. He maximizes expected utility, and his VNM utility function is \( u(w) = \sqrt{w} \).

   (a) What is the \( u''(w) \), i.e., the second derivative of this person’s utility function?
   (b) What is the expected amount of money he will lose? What is his expected wealth?
   (c) What is his expected utility?
   (d) What is his certainty equivalent wealth, i.e., the certain wealth level that gives him the same expected utility as his uncertain situation?
   (e) What is the maximum amount he would pay for full insurance, i.e., what is the maximum premium he would pay an insurance company to cover all of his losses?
   (f) What is his risk premium, i.e., how much more than his expected loss is he willing to pay for full insurance?
   (g) How do your answers to the above questions change if his utility function is \( u(w) = 3\sqrt{w} + 20 \) instead of \( u(w) = \sqrt{w} \). Explain.

2. An individual with initial wealth of $400 has a 20% chance of getting in an accident. If he gets in an accident, he will lose $300, leaving him with $100; if he doesn’t, he loses nothing. He maximizes expected utility, and his VNM utility function is \( u(w) = 20w + 10 \).

   (a) What is the \( u''(w) \), i.e., the second derivative of this person’s utility function?
   (b) What is the expected amount of money he will lose? What is his expected wealth?
   (c) What is his expected utility?
   (d) What is his certainty equivalent wealth?
   (e) What is the maximum amount he would pay for full insurance?
   (f) What is his risk premium, i.e., how much more than his expected loss is he willing to pay for full insurance?

3. An individual with initial wealth of $400 has a 20% chance of getting in an accident. If he gets in an accident, he will lose $300, leaving him with $100; if he doesn’t, he loses nothing. He maximizes expected utility, and his VNM utility function is \( u(w) = w^2 \).
(a) What is the \( u''(w) \), i.e., the second derivative of this person’s utility function?

(b) What is the expected amount of money he will lose? What is his expected wealth?

(c) What is his expected utility?

(d) What is his certainty equivalent wealth?

(e) What is the maximum amount he would pay for full insurance?

(f) What is his risk premium, i.e., how much more than his expected loss is he willing to pay for full insurance?

4. Consider an expected-utility–maximizing individual with VNM utility function \( u(w) = \sqrt{w} \) and initial wealth $400. With probability \( p \) she will get into an accident that will result in a loss of $300; with probability \((1 - p)\) she loses nothing. Imagine that she can buy insurance, but that the insurance company charges her an $8 application fee in addition to her expected loss.

(a) Draw a graph with \( w_1 \), her wealth if she doesn’t get into an accident, on the \( x \)-axis, and \( w_2 \), her wealth if she does get into an accident, on the \( y \)-axis. Then represent the following on the graph (not with mathematical precision, just with the ideas): her initial endowment point, indicating its coordinates; the indifference curve containing her endowment point; the fair-odds line for actuarially fair insurance (i.e., the set of all points that an actuarially fair policy would cover), indicating the equation for this line; and the fair-odds line for the insurance with the $8 application fee (i.e., the set of all points that could be covered with an $8 fee plus whatever her expected loss is), indicating the equation for this line. Indicate on the graph what distance(s), if any, represent(s) the $8 application fee.

(b) For what values of \( p \) will she purchase insurance? You should find that she will choose to self-insure for values of \( p \) close to either 0 or 1; provide an intuitive explanation for this result.

(c) For a random variable \( x \), the variance of \( x \), \( \text{Var}(x) \), is defined as

\[
\text{Var}(x) = E[(x - E(x))^2].
\]

Calculate the variance of this individual’s loss and the variance of her wealth (when you do this, you should assume that she does not have insurance); then compare the two; then note any apparent connections between these variances and her choice about whether or not to buy insurance for different values of \( p \).

(d) How do your answers to the above questions change if her utility function is \( u(w) = 3\sqrt{w} + 20 \) instead of \( u(w) = \sqrt{w} \). Explain.