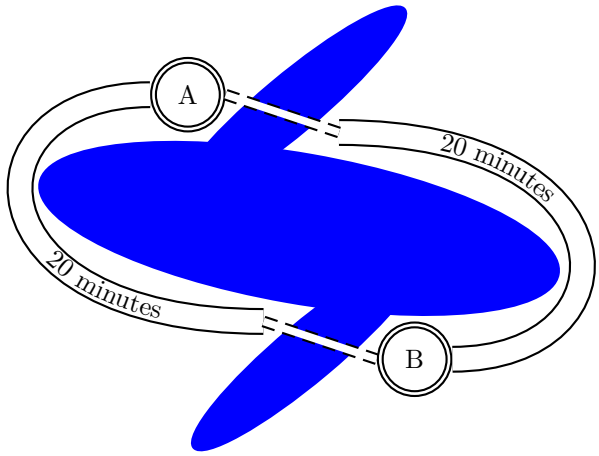
Braess's Traffic Paradox

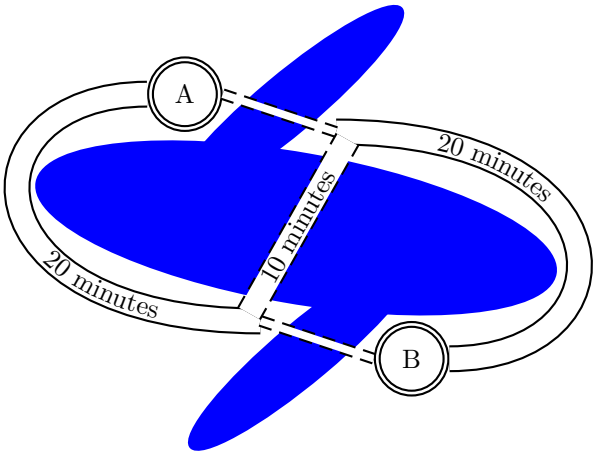
Twelve people must drive from point A to point B at the same time. As illustrated in the diagram below, there are two possible routes, each of which involves a highway and a bridge. The highways are wide and open and free, and the travel time on each highway—regardless of the number of vehicles—is 20 minutes. The bridges are small and congestible; for each traveller crossing a bridge, the number of minutes it takes to cross that bridge is equal to the total number of vehicles choosing to cross that bridge.



1. Find the Nash equilibrium(s) in this game. For each equilibrium, determine the total travel time for each driver.

Building a Shortcut

Now imagine that a shortcut is built as illustrated below. The shortcut is also a highway; travel time—regardless of the number of vehicles—is 10 minutes.



1. What happens to the Nash equilibrium(s) identified above?
2. Consider the action profile in which four drivers take each route (the left highway, the right highway, and the shortcut). What are the travel times for each group? Is this action profile a Nash equilibrium?
3. Consider the action profile in which eight drivers take the shortcut and two drivers take each of the other routes. What are the travel times for each group? Is this action profile a Nash equilibrium?
4. Is there a Nash equilibrium in which seven drivers take the shortcut? What about nine drivers?