Problem set due in class Thursday, April 28

Homework is graded check/check plus/check minus. You may work on these problems together, but you should write up your answers on your own. Note that problems marked *optional* are just that—optional—and that the author's website (linked from our class homepage) has answers to exercises with a dark circle around the question mark. **Please circle your answers and otherwise make it easy for me to follow your work.**

1. Exercise 192.1 (just draw what the game tree looks like and then find the SPNE).

2. Consider a repeated game in which the stage game is the Prisoners' Dilemma game below.

		Player 2	
		D	C
Player 1	D	0,0	3,-1
	C	-1,3	1,1

   (a) Argue that the only SPNE if the game is repeated finitely many times is for both players to always play D.

   (b) Argue that an SPNE is generated by the strategy pair in which both players always play D.

   (c) Consider the possibility that both players follow a "trigger strategy" whereby they play C as long as neither player has ever played D and otherwise play D. Find the values of δ for which this strategy pair forms an SPNE.

   (d) Modify the trigger strategy above to get a pair of trigger strategies that induces an outcome of $(C, C), (D, D), (C, C), \ldots$. (Make sure to formally define these strategies!) Find the values of δ for which this strategy pair forms an SPNE.

   (e) Modify the trigger strategy above to get a pair of trigger strategies that induces an outcome of $(C, D), (D, C), (C, D), \ldots$. (Make sure to formally define these strategies!) Find the values of δ for which this strategy pair forms an SPNE.

3. Consider a Stackelberg problem with two firms facing an inverse demand curve of $p = \alpha - q_1 - q_2$. Firm 1's costs are $C_1(q_1) = 0$ (i.e., zero costs) and firm 2's costs are $C_2(q_2) = \beta q_2$.

(a) Write down firm 2's optimization problem (with choice variables, objective function, and relevant constraints) and find firm 2's best response function to firm 1's choice of q_1. Make sure to consider corner solutions!

(b) Imagine that firm 2 didn't exist, so that firm 1 has a monopoly. Write down firm 1's optimization problem (with choice variables, objective function, and relevant constraints) and find the value of q_1 that maximizes firm 1's profits. Call this the value of q_1 the "ideal monopoly output."

(c) Returning to the Stackelberg game, write down firm 1's optimization problem (with choice variables, objective function, and relevant constraints).

(d) (Passive deterrence) If α and β are "very close," the SPNE in this game generates an outcome in which firm 1 produces $q_1 = \frac{\alpha}{2}$, the ideal monopoly output, and firm 2 produces $q_2 = 0$. Show that this is true for the specific example of $\alpha = 15, \beta = 10$, and formally describe the SPNE in this case. (Hint: To tackle this problem, first argue that firm 2 will produce $q_2 = 0$ if firm 1 produces the ideal monopoly output; then make a logical argument that this must be the outcome that maximizes profit for firm 1.)

(e) (Active deterrence) If α and β are "sort-of close," the SPNE in this game produces an outcome in which firm 1 produces *more* than the ideal monopoly output and firm 2 produces $q_2 = 0$. Show that this is true for the specific example of $\alpha = 30, \beta = 10$, and formally describe the SPNE in this case. (Hint: To tackle this problem, first find firm 1's maximum profit if it chases firm 2 out of the market, i.e., produces enough so that $q_2 = 0$ is the best response; then try to find firm 1's maximum profit if it accommodates firm 2, i.e., does not produce enough to force $q_2 = 0$ (you will find that firm 1's profits increase as q_2 approaches zero); and conclude that firm 1 maximizes profits by forcing firm 2 out of the market.)

(f) (Accommodation) If α and β are "not close," the SPNE in this game produces an outcome in which both firms produce non-zero amounts. Show that this is true for the specific example of $\alpha = 90, \beta = 10$, and formally describe the SPNE in this case. (Hint: To tackle this problem, first find firm 1's maximum profit if it chases firm 2 out of the market, i.e., produces enough so that $q_2 = 0$ is the best response; then find firm 1's maximum profit if it accommodates firm 2, i.e., does not produce enough to force $q_2 = 0$; then show that firm 1's profits are higher if it accommodates firm 2.)

4. Read the sidebars on repeated Prisoners' Dilemma games (pp. 436–437, 439–442, and 448–449) and write a one-sentence comment on each.