Algorithms for Finding Nash Equilibria

The best way to identify the Nash equilibria of a game is to first identify all of the outcomes that are *not* Nash equilibria; anything left must be a Nash equilibrium. For example, consider the game in Figure 1. The strategy pair (U, L) is not a Nash equilibrium because Player 2 can gain by deviating alone to R; (U, C) is not a NE because Player 1 can gain by deviating alone to D (and Player 2 can gain by deviating alone to L or R); etc. If you go through the options one by one and cross out those that are *not* Nash equilibria, the remaining options *will* be Nash equilibria (See Figure 2a).

Player 2

	L	C	R
U	5,1	2,0	2,2
M	0,4	1,5	4,5
D	2,4	3,6	1,0

Player 1 is on the left (U, M, D rows).

Figure 1: Nash equilibrium example

A shortcut (but one you should use carefully!) is to underline each player's best responses.[1] To apply this to the game in Figure 1, first assume that Player 2 plays L; Player 1's best response is to play U, so underline the "5" in the box corresponding to (U, L). Next assume that Player 2 plays C; Player 1's best response is to play D, so underline the "3" in the box corresponding to (D, C). Finally, assume that Player 2 plays R; Player 1's best response is to play M, so underline the "4" in the box corresponding to (M, R). Now do the same thing for Player 2: go through all of Player 1's options and underline the best response for Player 2. (Note that C and R are both best responses when Player 1 plays M!) We end up with Figure 2b: the only boxes with both payoffs underlined are (D, C) and (M, R), the Nash equilibria of the game.

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[1]It is easy to confuse the rows and columns and end up underlining the wrong things. Always double-check your answers by confirming that no player can gain by deviating alone.

	L	C	R			L	C	R
U	5̶,1̶	2̶,0̶	2̶,2̶		U	5̲,1	2,0	2,2̲
M	0̶,4̶	1̶,5̶	4,5		M	0,4	1,5̲	4̲,5̲
D	2̶,4̶	3,6	1̶,0̶		D	2,4	3̲,6̲	1,0

(a)   (b)

Figure 2: Finding Nash equilibria: (a) with strike-outs; (b) with underlinings