(5 points) Name:

Final Exam (75 Points Total)

1. Consider a product for which demand is given by $q = 100 - p$ for $p < 100$ (and $q = 0$ for $p \geq 100$) and for which there are two competing producers. Firm 1 has zero costs, i.e., $C_1(q_1) = 0$. Firm 2 has zero fixed costs and marginal costs of \$10, i.e., $C_2(q_2) = 10q_2$.

   (a) (5 points) Show that Firm 1 would choose a price of $p_1 = \$50$ if it were a monopoly. Make sure to clearly identify the choice variable(s), the objective function, and whatever constraints may apply, and make sure to check corner solutions.

   (b) (5 points) Imagine that the two firms compete in a Bertrand game *with continuous prices* $p_1 \geq 0$ and $p_2 \geq 0$. (In this game, the firm with the lower cost wins the entire market; in the case of a tie, the firms share the market.) Show that there is no Nash equilibrium in this game. (Hint: Show that there is no Nash equilibrium (p_1, p_2) in any of the following situations: first, $p_2 = 0$; second, $0 < p_2 \leq 50$; third, $p_2 > 50$ and $p_1 \neq 50$; and fourth, $p_2 > 50$ and $p_1 = 50$.)

(c) (5 points) Imagine that the two firms compete in a Bertrand (price competition) game *with discrete prices*, meaning that prices are limited to dollars and cents, e.g., \$9.99. Find Firm 1's best response function to Firm 2's choice of p_2. (Hint: Make sure to consider the result of question 1a, and make sure to indicate Firm 1's best response to $p_2 = 0$.)

(d) (5 points) Find the Nash equilibrium(s) in this game with discrete prices.

(e) (5 points) Imagine that the two firms compete in a Cournot (quantity competition) game, so that the market price is $p = 100 - q_1 - q_2$. (As usual in Cournot games, the choices of q_1 and q_2 are *not* limited to discrete quantities.) Write down the maximization problems facing Firm 1 and Firm 2, making sure to clearly identify the choice variables, the objective functions, and whatever constraints may apply. Then explain how to find an interior Nash equilibrium for this Cournot problem. (You do not have to actually solve the problem.)

(f) (5 points) Find the best response functions for Firm 1 and Firm 2 in this game.

(g) (5 points) Imagine that the two firms compete in a Stackelberg (leader-follower) game, so that the market price is $p = 100 - q_1 - q_2$ and Firm 1 chooses q_1 before Firm 2 chooses q_2. Write down the maximization problems facing Firm 1 and Firm 2 for a subgame perfect Nash equilibrium, making sure to clearly identify the choice variables, the objective functions, and whatever constraints may apply. *You do not have to actually solve the problem, but you should make sure to indicate how your answer here differs from your answer to the Cournot game above.*

(h) (5 points) Solve the Stackelberg problem by identifying the SPNE for this game. (You should assume that there is an interior solution.) What outputs q_1 and q_2 will we actually see in this game?

(i) (5 points) If Firm 1 produces $q_1 = 0$ then Firm 2's best response is to produce its monopoly output, which turns out to be $q_2 = 45$. Can you identify a Nash equilibrium in which Firm 1's strategy is to produce $q_1 = 0$? (Hint: Such a NE does not have to be—and in fact cannot be—subgame perfect, so think about "punishments" that Firm 2 can use if Firm 1 produces $q_1 \neq 0$.) Also: explain what is "wrong" with this NE, i.e., why it is important to focus on subgame perfect Nash equilibriums.

2. (5 points) Write down a two-player Prisoners' Dilemma situation and identify all the Nash equilibrium (in pure and mixed strategies).

Player 2

		U	D
Player 1	U	2,2	1,3
	D	3,1	0,0

Figure 1: A simultaneous move game.

3. Consider the game shown above.

   (a) (5 points) Determine the best response function for Player 1 if p and q represent the probability that Players 1 and 2, respectively, will play U. (Note, for the problem below, that Player 2 has a symmetric best response function.)

   (b) (5 points) Identify all the Nash equilibrium (in pure and mixed strategies) in this game. (Hint: You should find a mixed strategy Nash equilibrium at $p = q = \frac{1}{2}$. If that's not what you got above, show that $p = q = \frac{1}{2}$ is a Nash equilibrium of this game by checking to see if any player can gain by deviating alone.)

		Player 2	
		U	D
Player 1	U	2,2	1,3
	D	3,1	0,0

Figure 2: The same game.

(c) (5 points) The next question asks you to determine if $p = q = \frac{1}{2}$ is an evolutionarily stable strategy. First, however, you should provide an intuitive explanation of what an evolutionarily stable strategy is. You should also provide a clear explanation of how to interpret evolutionarily stable *mixed* strategies. (There are two such interpretations. You only need to give one.)

(d) (5 points) Is $p = q = \frac{1}{2}$ an evolutionarily stable strategy? Clearly indicate why or why not.