Exam #1 (100 Points Total) **Answer Key**

1. (a) Firm 1's profits are

$$\pi_1 = p_1 q_1 - C_1(q_1) = (10 - 2q_1 - q_2)q_1 - 3q_1^2 = (10 - q_2)q_1 - 5q_1^2.$$

Firm 2's profits are

$$\pi_2 = p_2 q_2 - C_2(q_2) = (10 - q_1 - 2q_2)q_2 - 3q_2^2 = (10 - q_1)q_2 - 5q_2^2.$$

With collusion, the firms choose q_1 and q_2 to maximize joint profits

$$\pi_1 + \pi_2 = (10 - q_2)q_1 - 5q_1^2 + (10 - q_1)q_2 - 5q_2^2.$$

To solve this problem, we take partial derivatives with respect to each choice variable and set them equal to zero. This will give us two necessary first-order conditions (NFOCs) in two unknowns (q_1 and q_2); solving these simultaneously gives us our optimum.
So: the NFOCs are

$$\frac{\partial(\pi_1 + \pi_2)}{\partial q_1} = 0 \Longrightarrow 10 - q_2 - 10q_1 - q_2 = 0$$

and

$$\frac{\partial(\pi_1 + \pi_2)}{\partial q_2} = 0 \Longrightarrow -q_1 + (10 - q_1) - 10q_2 = 0$$

Solving these jointly yields $q_1 = q_2 = \frac{10}{12} \approx .83$. The prices are therefore $p_1 = p_2 \approx 10 - 3(.83) \approx 7.51$ and industry profits are

$$\pi_1 + \pi_2 = 2\left(p_1 q_1 - C(q_1)\right) \approx 2\left(7.51(.83) - 3(.83^2)\right) \approx 4.17.$$

(b) Take partial derivatives with respect to all the choice variables, set them equal to zero, and solve the resulting equations simultaneously to find the interior solutions.

(c) Here Firm 1 chooses q_1 to maximize its profits and Firm 2 chooses q_2 to maximize its profits. (The profit functions are given above.) To solve this problem we take a partial derivative of π_1 with respect to q_1 to get a necessary first-order condition (NFOC) for Firm 1. We then take a partial derivative of π_2 with respect to q_2 to get a necessary first-order condition (NFOC) for Firm 2. Solving these NFOCs simultaneously gives us the Cournot outcome.
So: the NFOCs are

$$\frac{\partial(\pi_1)}{\partial q_1} = 0 \Longrightarrow 10 - q_2 - 10q_1 = 0$$

and

$$\frac{\partial(\pi_2)}{\partial q_2} = 0 \Longrightarrow (10 - q_1) - 10q_2 = 0$$

Solving these jointly yields $q_1 = q_2 = \frac{10}{11} \approx .91$. The prices are therefore $p_1 = p_2 \approx 10 - 3(.91) = 7.27$.

(d) Take partial derivatives of each objective function with respect to each choice variable, set them equal to zero, and solve the resulting equations simultaneously to find the interior solutions.

2. (a) The two pure strategy Nash equilibriums are (U, R) and (D, L).

(b) Player 1 chooses $p, 0 \leq p \leq 1$, to maximize

$$\pi_1 = pq(0)+p(1-q)(2)+(1-p)q(1)+(1-p)(1-q)(0) = 2p+q-3pq.$$

At a maximum, either $p = 0$ or $p = 1$ or there is an interior solution, $0 < p < 1$, in which case

$$\frac{\partial \pi_1}{\partial p} = 0 \Longrightarrow 2 - 3q = 0 \Longrightarrow q = \frac{2}{3}.$$

Plugging $q = \frac{2}{3}$ into Player 1's objective function shows that $\pi_1 = \frac{2}{3}$ regardless of the choice of p, which means that any $p, 0 \leq p \leq 1$, is a best response if $q = \frac{2}{3}$. If $q \neq \frac{2}{3}$ then Player 1's best response is a corner solution, either $p = 0$ or $p = 1$. If $p = 0$ then $\pi_1 = q$, and if $p = 1$ then $\pi_1 = 2 - 2q$, so we can see that the rest of Player 1's best response function is to choose $p = 1$ if $q < \frac{2}{3}$ and to choose $p = 0$ if $q > \frac{2}{3}$.

(c) Player 2 chooses $q, 0 \leq q \leq 1$, to maximize

$$\pi_2 = q(1) + p(1-q)(2) + (1-p)(1-q)(0) = 2p + q - 2pq.$$

At a maximum, either $q = 0$ or $q = 1$ or there is an interior solution, $0 < q < 1$, in which case

$$\frac{\partial \pi_2}{\partial q} = 0 \Longrightarrow 1 - 2p = 0 \Longrightarrow p = \frac{1}{2}.$$

Plugging $p = \frac{1}{2}$ into Player 2's objective function shows that $\pi_2 = 1$ regardless of the choice of q, which means that any $q, 0 \leq q \leq 1$, is a best response if $p = \frac{1}{2}$. If $p \neq \frac{1}{2}$ then Player 2's best response is a corner solution, either $q = 0$ or $q = 1$. If $q = 0$ then $\pi_2 = 2p$, and if $q = 1$ then $\pi_2 = q$, so we can see that the rest of Player 2's best response function is to choose $q = 1$ if $p < \frac{1}{2}$ and to choose $q = 0$ if $p > \frac{1}{2}$.

(d) There are three Nash equilibriums. One is given by $p = 1, q = 0$, and a second is given by $p = 0, q = 1$; these correspond to the pure strategy Nash equilibriums found above. The third Nash equilibrium is a non-degenerate mixed strategy Nash equilibrium given by $p = \frac{1}{2}, q = \frac{2}{3}$.

3. (a) Any $b_1 \geq 0$ is a best response, so $b_1 = v$ is indeed a best response.

   (b) Any $b_1 < 1000 - b_2 - b_3$ is a best response, and since $v < 1000 - b_2 - b_3$ it follows that $b_1 = v$ is a best response.

   (c) Any $b_1 \geq 1000 - b_2 - b_3$ is a best response, and since $v \geq 1000 - b_2 - b_3$ it follows that $b_1 = v$ is a best response.

   (d) You have a strictly dominant strategy if pursuing that strategy always gives you a strictly higher payoff than any other action you could choose, regardless of what the other players do; alternately, a strictly dominant strategy is *the* (unique) best response to whatever actions the other players choose.

   (e) In this game you have no strictly dominant strategy. If, for example, another player bids \$2000 then your payoff will be the same regardless of what you bid.

   (f) You have a weakly dominant strategy if pursuing that strategy always gives you at least as high a payoff as any other action you could choose, regardless of what the other players do; alternately, a weakly dominant strategy is *a* (not necessarily unique) best response to whatever actions the other players choose.

   (g) In this game, bidding your true value is a weakly dominant strategy.

   (h) A Nash equilibrium occurs when the players' strategies are mutual best responses.

   (i) One Nash equilibrium in this game occurs when each player bids their true value. As the next problem shows, there are many others.

   (j) If multiple players submit extremely high bids, none of them will be decisive and consequently none of them will have to pay the special tax.

   (k) You would get a Nash equilibrium if, say, all three players bid \$2000. No player can gain by deviating alone because the outcome is independent of the bid of any one player.