(5 points) Name:

Exam #1 (100 Points Total)

1. Consider a model with two firms, each with costs $C_i(q_i) = 3q_i^2$. The firms produce similar but not identical goods (think Coke and Pepsi, or other examples of monopolistic competition), meaning that the demand curves they face are related but not identical. In particular, the inverse demand curve for Firm 1's output is $p_1 = 10 - 2q_1 - q_2$ for $2q_1 + q_2 < 10$ and $p_1 = 0$ otherwise; and the inverse demand curve for Firm 2's output is $p_2 = 10 - q_1 - 2q_2$ for $q_1 + 2q_2 < 10$ and $p_2 = 0$ otherwise.

   (a) (5 points) Imagine that the same parent company owns both firms and has a goal of maximizing joint profits. (Maximization of joint profits is a common goal in such collusive situations.) Write down the optimization problem for the parent company, making sure to clearly identify the choice variable(s), the objective function, and whatever constraints may apply.

   (b) (5 points) Describe how to go about finding an interior solution to this problem.

(c) (5 points) Now imagine that the two firms are engaged in Cournot-style quantity competition. Write down the optimization problem for Firm 1, making sure to clearly identify the choice variable(s), the objective function, and whatever constraints may apply.

(d) (5 points) Assuming that you have a similar optimization problem for Firm 2, describe how to go about finding an interior solution to this problem.

2. Consider the game shown in the figure below.

(a) (5 points) Identify (e.g., by circling) the pure strategy Nash equilibrium(s), if any, of this game.

		Player 2	
		L	R
Player 1	U	0,1	2,2
	D	1,1	0,0

Figure 1: A simultaneous move game.

Player 2

		L	R
Player 1	U	0,1	2,2
	D	1,1	0,0

Figure 2: The same game.

(b) (5 points) Determine the best response function for Player 1.

(c) (5 points) Determine the best response function for Player 2.

Player 2

		L	R
Player 1	U	0,1	2,2
	D	1,1	0,0

Figure 3: The same game, again.

(d) (5 points) Find all the mixed strategy Nash equilibriums in this game.

3. Imagine that you and two other people live in a town whose government is trying to decide whether or not to spend \$1000 to create a new park. It only wants to do so if the value that citizens will get from the park (as measured by their willingness-to-pay) is at least \$1000, but the government can't just go around and ask people how much they'd be willing to pay for the park: some people might be tempted to lie in order to influence the outcome. So the government decides to ask you (and the other two people) to submit bids of $b_1 \geq 0, b_2 \geq 0$, and $b_3 \geq 0$, respectively, and announces the following policy: if the bids don't add up to at least \$1000 (i.e., if $b_1 + b_2 + b_3 < 1000$) then the park doesn't get built; if the bids add up to over \$1000 *and your bid is decisive* (i.e., if $b_1 + b_2 + b_3 \geq 1000$ but $b_2 + b_3 < 1000$), then the park gets built and you pay a special tax of $1000 - b_2 - b_3$; if the bids add up to over \$1000 *and your bid is not decisive* (i.e., if $b_2 + b_3 \geq 1000$), then the park gets built and you pay no additional tax.

   (a) (5 points) Determine the bid(s) $b_1 \geq 0$ that constitute(s) your best response(s) if $b_2 + b_3 \geq 1000$. Is bidding $b_1 = v$, where v is your true value, a best response?

   (b) (5 points) Determine the bid(s) $b_1 \geq 0$ that constitute(s) your best response(s) if $b_2 + b_3 < 1000$ and $b_2 + b_3 + v < 1000$, i.e., if the only way to get a sum of at least \$1000 is to bid more than your true value. Is $b_1 = v$ a best response? (Hint: Do you want the park to get built in this case? What bids will ensure that that will or won't happen?)

   (c) (5 points) Determine the bid(s) $b_1 \geq 0$ that constitute(s) your best response(s) if $b_2 + b_3 < 1000$ but $b_2 + b_3 + v \geq 1000$, i.e., if you don't have to bid more than your true value to get a sum of at least \$1000. Is $b_1 = v$ a best response? (Hint: Again, think about whether or not you want the park to get built.)

(d) (5 points) Define (in English) what it would mean for you to have a strictly dominant strategy in this game.

(e) (5 points) Do you have a strictly dominant strategy in this game? If so, identify it. If not, explain why not.

(f) (5 points) Define (in English) what it would mean for you to have a weakly dominant strategy in this game.

(g) (5 points) Do you have a weakly dominant strategy in this game? If so, identify such a strategy. If not, explain why not.

(h) (5 points) Define (in English) what it would mean for the players' strategies to form a Nash equilibrium.

(i) (5 points) Identify a Nash equilibrium in this game that builds off your answer to the first three questions above.

(j) (5 points) Describe how the players could work together to "game" the system in such a way that they could guarantee that the park would get built without any of them having to worry about paying the special tax.

(k) (5 points) Identify a Nash equilibrium in this game that builds off your answer to the previous question.