1 Duopoly

Returning to a world without price discrimination, we can now consider what happens in a market with two producers (i.e., a **duopoly**). We will study three different scenarios: collusion (in which the firms work together), Cournot competition (in which the firms compete on the basis of quantity), and Stackleberg competition (in which one firm produces and then the other firm responds.)

The situation we will consider is the following: Coke and Pepsi are engaged in monopolistic competition in the diet soft drink industry. Both firms have costs $C(q) = 5q$, where q is measured in millions of gallons of syrup. Coke can set any price p_C for each gallon of its syrup, and Pepsi can similarly choose p_P, but both are constrained by their customers' demand curves. Since the two products are substitutes, their demand curves are interrelated, as indicated by the inverse demand curves

$$\begin{aligned} p_C &= 20 - 2q_C - q_P \\ p_P &= 20 - 2q_P - q_C. \end{aligned}$$

These inverse demand curves indicate that the maximum price that Coke can charge is strongly influenced by the amount of Diet Coke it produces, and less strongly influenced by the amount of Diet Pepsi that Pepsi produces. The same is true for Pepsi: the more Diet Pepsi they produce, the less they can charge; and the more Diet Coke Coke produces, the less Pepsi can charge.

Each company wants to maximize profits subject to the constraints of their respective demand curves. As in the monopoly situation above, we can substitute for the per-gallon prices p_C and p_P from the inverse demand curves to get the following objective functions and choice variables:

Coke wants to choose q_C to maximize $\pi_C = p_C(q_C, q_P) \cdot q_C - C(q_C)$.

Pepsi wants to choose q_P to maximize $\pi_P = p_P(q_C, q_P) \cdot q_P - C(q_P)$.

These objective functions simplify as

$$\begin{aligned} \pi_C &= (20 - 2q_C - q_P) \cdot q_C - 5q_C = (15 - q_P)q_C - 2q_C^2 \\ \pi_P &= (20 - 2q_P - q_C) \cdot q_P - 5q_P = (15 - q_C)q_P - 2q_P^2. \end{aligned}$$

Collusion

In the collusive situation the companies work together to maximize their *joint* profits. In other words, we imagine a situation in which a single firm (say, Microsoft) buys both Coke *and* Pepsi, somehow miraculously avoiding antitrust scrutiny. In this case the owner has two choice variables (q_C and q_P) and chooses them to maximize joint profits,

$$\pi_C + \pi_P = (15 - q_P)q_C - 2q_C^2 + (15 - q_C)q_P - 2q_P^2.$$

Our necessary first-order condition (NFOC) for an interior maximum is that the partial derivatives with respect to the choice variables must all equal zero, so we must have

$$\frac{\partial(\pi_C + \pi_P)}{\partial q_C} = 0 \Longrightarrow 15 - q_P - 4q_C - q_P = 0 \Longrightarrow 4q_C = 15 - 2q_P$$

and

$$\frac{\partial(\pi_C + \pi_P)}{\partial q_P} = 0 \Longrightarrow -q_C + 15 - q_C - 4q_P = 0 \Longrightarrow 4q_P = 15 - 2q_C.$$

We now have a system of two simultaneous equations ($4q_C = 15 - 2q_P$ and $4q_P = 15 - 2q_C$) in two unknown (q_C and q_P). We can solve these, e.g., by solving the first equation for q_C and plugging into the second:

$$4q_P = 15 - 2q_C \Longrightarrow 4q_P = 15 - 2\left(\frac{15 - 2q_P}{4}\right) \Longrightarrow 8q_P = 30 - 15 + 2q_P.$$

This simplifies to $q_P = 2.5$, and we can plug this back in to either of our NFOCs to get $q_C = 2.5$.

So the output choices that maximize joint profits are $q_C = q_P = 2.5$, and we can plug these numbers into the joint profit function to get

$$\pi_C + \pi_P = (15 - q_P)q_C - 2q_C^2 + (15 - q_C)q_P - 2q_P^2 = 37.5.$$

So if the two firms collude then they will each produce 2.5 million gallons of syrup and will have a combined profit of \$37.5 million.

We can also use the inverse demand curves to find the prices they will charge, e.g., $p_C = 20 - 2q_C - q_P = 12.5$ and $p_P = 20 - 2q_P - q_C = 12.5$; so each firm will charge \$12.50 for each gallon of syrup. To check our answers, we can recalculate profits (hopefully yielding 37.5) using

$$\pi_C + \pi_P = p_C \cdot q_C - 2q_C^2 + p_P \cdot q_P - 2q_P^2.$$

Cournot (Quantity) Competition

Our next scenario features what is called **Cournot competition**: the two firms simultaneously choose quantities q_C and q_P, the prices follow from the inverse demand curves, and profits are determined accordingly.

In this case there is *no* joint profit maximization. Instead, Coke chooses its quantity q_C to maximize its profits,

$$\pi_C = (20 - 2q_C - q_P) \cdot q_C - 5q_C = (15 - q_P)q_C - 2q_C^2$$

and Pepsi chooses its quantity q_P to maximize its profits,

$$\pi_P = (20 - 2q_P - q_C) \cdot q_P - 5q_P = (15 - q_C)q_P - 2q_P^2.$$

The necessary first-order condition (NFOC) for an interior maximum to Coke's problem is that the derivative with respect to its choice variable must be zero, i.e.,

$$\frac{d\pi_C}{dq_C} = 0 \Longrightarrow 15 - q_P - 4q_C = 0 \Longrightarrow q_C = 3.75 - .25q_P.$$

This is the **best response function** for Coke. *If* Pepsi produces q_P, *then* Coke's profit-maximizing choice of q_C is $q_C = 3.75 - .25q_P$: if Pepsi produces $q_P = 2$ then Coke's best response is $q_C = 3.25$; if Pepsi produces $q_P = 4$ then Coke's best response is $q_C = 2.75$; if Pepsi produces $q_P = 8$ then Coke's best response is $q_C = 1.75$; and if Pepsi produces $q_P = 16$ then Coke's best response is $q_C = -.25$, suggesting that we have a corner solution of $q_C = 0$. (If Pepsi produces $q_P = 16$ then Coke's inverse demand curve shows that Coke cannot charge a price higher than 4, in which case it cannot cover its production costs and should therefore produce 0.)

Similarly, the necessary first-order condition (NFOC) for an interior maximum to Pepsi's problem is that the derivative with respect to its choice variable must be zero, i.e.,

$$\frac{d\pi_P}{dq_P} = 0 \Longrightarrow 15 - q_C - 4q_P = 0 \Longrightarrow q_P = 3.75 - .25q_C.$$

This is the **best response function** for Pepsi. *If* Coke produces q_C, *then* Pepsi's profit-maximizing choice of q_P is $q_P = 3.75 - .25q_C$.

The Cournot solution to this game is to find mutual best responses, i.e., a Nash equilibrium solution. (Since Cournot solved this problem before Nash was born, it doesn't seem fair to call it the Nash equilibrium solution; it is, however, sometimes called the Nash-Cournot solution.)

To find the Cournot solution we simultaneously solve our two best response functions, $q_C = 3.75 - .25q_P$ and $q_P = 3.75 - .25q_C$. We can do this by plugging q_C into the second function and solving:

$$q_P = 3.75 - .25(3.75 - .25q_P) \Longrightarrow 4q_P = 15 - (3.75 - .25q_P) \Longrightarrow 3.75q_P = 11.25$$

This simplifies to $q_P = 3$; plugging into Coke's best response function we get $q_C = 3$.

So the outputs $q_C = 3$ and $q_P = 3$ are mutual best responses: if Coke produces 3 million gallons of Diet Coke, Pepsi's best response (i.e., its profit-maximizing choice) is to produce 3 million gallons of Diet Pepsi. And if Pepsi produces 3 million gallons of Diet Pepsi, Coke's best response is to produce 3 million gallons of Diet Coke.

We have therefore found the Cournot solution to this game: $q_C = q_P = 3$. We can now plug this answer into the various profit functions to get

$$\pi_C = (15 - q_P)q_C - 2q_C^2 = 18$$

and

$$\pi_P = (15 - q_C)q_P - 2q_P^2 = 18.$$

We can also solve for their respective prices from the inverse demand curves to get $p_C = p_P = 11$.

How does this compare with the cooperative outcome? Well, cooperating yields joint profits of \$37.5 million; under Cournot competition, the joint profits are only \$18 + \$18 = \$36 million. So the firms could do better by cooperating, meaning that firms would be likely to collude (e.g., by fixing prices) if it weren't illegal and if it were easy for them to communicate and write enforceable contracts.

2 The Transition to Perfect Competition

What happens if we take a monopoly model and add more firms? Well, if we add one firm we get duopoly. If we add additional firms we get **oligopoly**. The interesting thing about oligopoly is that it bridges the gap between monopoly (a market with just one firm) and competitive markets (a market with a large number of small firms).

To see how this works, consider a simple monopoly model: only one firm produces soda, the demand curve for soda is $q = 10-p$, and the cost of producing soda is $C(q) = 2q$. The monopolist chooses p and q to maximize $\pi = pq - C(q) = pq - 2q$ subject to the constraint $q = 10 - p$. Equivalently, the monopolist chooses q to maximize $\pi = (10 - q)q - 2q = 8q - q^2$. Setting a derivative equal to zero gives us $8 - 2q = 0$, meaning that the monopolist will choose $q = 4$, $p = 10 - q = 6$, and get profits of $\pi = pq - C(q) = 16$.

Next: imagine that a second firm, also with costs $C(q) = 2q$, enters the market and engages in Cournot competition with the first firm. The demand curve is still $q = 10 - p$, but now $q = q_1 + q_2$, i.e., the market output is the sum of each firm's output. We can transform the demand curve $q_1 + q_2 = 10 - p$ into the inverse demand curve, $p = 10 - q_1 - q_2$.

Now, firm 1 chooses q_1 to maximize $\pi_1 = pq_1 - C(q_1) = (10-q_1-q_2)q_1 - 2q_1 = (8-q_2)q_1 - q_1^2$. Setting a derivative equal to zero gives us $8 - q_2 - 2q_1 = 0$, which rearranges as $q_1 = 4 - .5q_2$. This is the best response function for firm 1: given q_2, it specifies the choice of q_1 that maximizes firm 1's profits.

Since the problem is symmetric, firm 2's best response function is $q_2 = 4 - .5q_1$. Solving these simultaneous to find the Cournot solution yields

$$q_1 = 4 - .5(4 - .5q_1) \Longrightarrow .75q_1 = 2 \Longrightarrow q_1 = \frac{8}{3} \approx 2.67.$$

We get the same result for q_2, so the market price will be $10 - q_1 - q_2 = \frac{14}{3} \approx 4.67$. Each firm will earn profits of

$$pq_i - 2q_i = \frac{14}{3} \cdot \frac{8}{3} - 2 \cdot \frac{8}{3} = \frac{64}{9} \approx 7.11,$$

so industry profits will be about $2(7.11) = 14.22$.

Next: imagine that a third firm, also with costs $C(q) = 2q$, enters the market and engages in Cournot competition with the first two firms. The demand curve is still $q = 10 - p$, but now $q = q_1 + q_2 + q_3$. We can transform the demand curve $q_1 + q_2 + q_3 = 10 - p$ into the inverse demand curve, $p = 10 - q_1 - q_2 - q_3$.

Now, firm 1 chooses q_1 to maximize $\pi_1 = pq_1 - C(q_1) = (10 - q_1 - q_2 - q_3)q_1 - 2q_1 = (8 - q_2 - q_3)q_1 - q_1^2$. Setting a derivative equal to zero gives us $8 - q_2 - q_3 - 2q_1 = 0$, which rearranges as $q_1 = 4 - .5(q_2 + q_3)$. This is the best response function for firm 1: given q_2 and q_3, it specifies the choice of q_1 that maximizes firm 1's profits.

Since the problem is symmetric, firms 2 and 3 have similar best response functions, $q_2 = 4 - .5(q_1 + q_3)$ and $q_3 = 4 - .5(q_1 + q_2)$, respectively. Solving these three best response functions simultaneous yields $q_1 = q_2 = q_3 = 2$. (Note: the brute force method of solving these equations simultaneous gets a bit messy. An easier way is to notice that the solution must be symmetric, i.e., with $q_1 = q_2 = q_3$; substituting for q_2 and q_3 in firm 1's best response function yields $q_1 = 4 - .5(2q_1)$, which quickly leads to the solution.)

So with three firms we have each firm producing $q_i = 2$; output price will therefore be $p = 10 - q_1 - q_2 - q_3 = 4$, and each firm will have profits of $\pi_i = pq_i - C(q_i) = 4(2) - 2(2) = 4$. Industry profits will therefore be $3(4) = 12$.

If we step back, we can see a general trend: the total amount produced by the industry is increasing (from 4 to $2(2.67) = 5.34$ to $3(2) = 6$), the price is dropping (from 6 to 4.67 to 4), and total industry profits are decreasing (from 16 to 14.22 to 12).

We can generalize this by considering a market with n identical firms, each with costs $C(q) = 2q$. Facing a demand curve of $q_1 + q_2 + \ldots + q_n = 10 - p$, we can get an inverse demand curve of $p = 10 - q_1 - q_2 - \ldots - q_n$. Firm 1, for example, will then choose q_1 to maximize

$$\pi_1 = pq_1 - C(q_1) = (10 - q_1 - q_2 - \ldots - q_n)(q_1) - 2q_1 = (8 - q_2 - q_3 - \ldots - q_n)q_1 - q_1^2.$$

Setting a derivative equal to zero gives us a necessary first order condition (NFOC) of $8 - q_2 - q_3 - \ldots - q_n - 2q_1 = 0$. The other firms will have symmetric solutions, so in the end we must have $q_1 = q_2 = \ldots = q_n$. Substituting these into the NFOC gives us

$$8 - (n-1)q_1 - 2q_1 = 0 \Longrightarrow 8 - (n+1)q_1 = 0 \Longrightarrow q_1 = \frac{8}{n+1}.$$

Each firm produces the same amount of output, so total industry output will be $\frac{8n}{n+1}$. As n gets larger and larger, approaching infinity, total industry output therefore approaches 8. The market price for any given n is $p = 10 - \frac{8n}{n+1}$, so as n approaches infinity the market price approaches $10 - 8 = 2$. Since production costs are $C(q) = 2q$, this means that firm profits approach zero as the number of firms gets larger and larger. We will see in the next part of the course that the industry is converging to the competitive market outcome! So competitive markets can be thought of as a limiting case of oligopoly as the number of firms gets very large. . . .